A Water Bag Model of Phase Space Holes in Nonneutral Plasma

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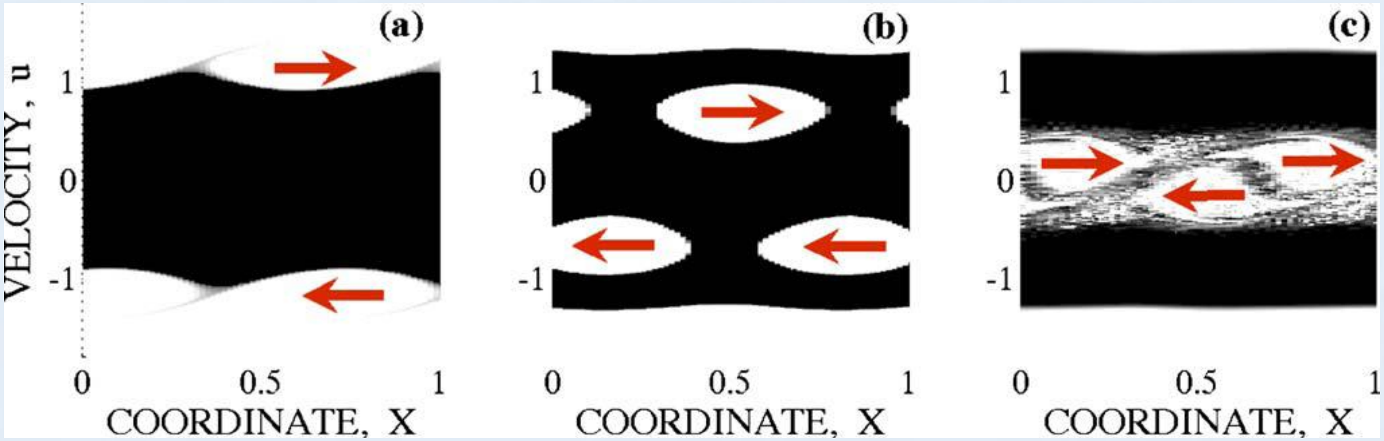
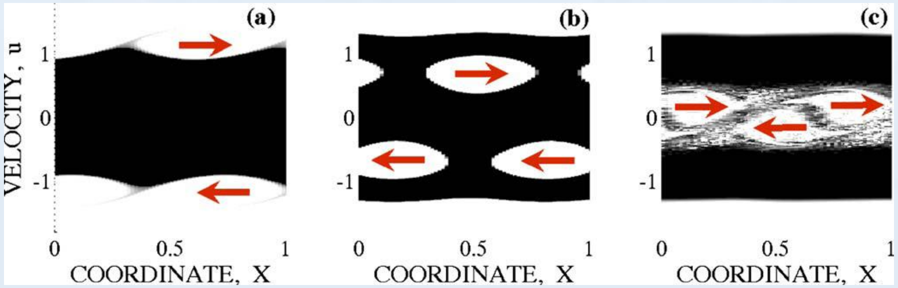


I. Barth, L. Friedland, and A.G. Shagalov, Phys. Plasmas, 15, 082110 (2008)

BGK (Berstein, Greene and Kruskal) modes

- Electrostatic, dissipationless (no Landau damping) waves
- Fluid-type description is not sufficient (resonant particles).
- Need of kinetic theory (Vlasov-Poisson)
- Observed:
  - plasma experiments , but difficult to control
  - Magnetosphere
  - Solar wind
  - Supernova remnant

Phase space holes ⟷ BGK mode



How to excite and control BGK modes ?

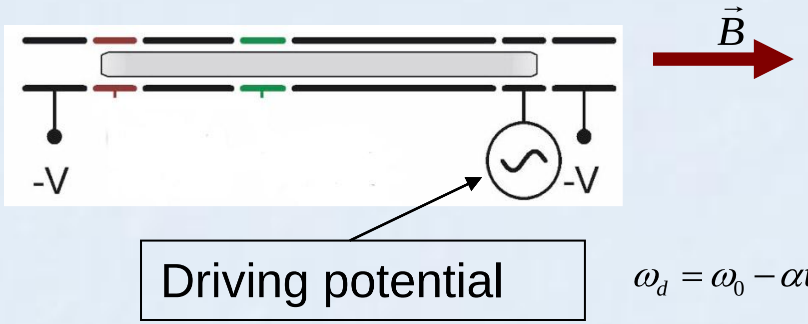
Our approach:

- Take a Penning trapped pure-ion plasma.
- Apply an oscillating external potential.
- Slowly reduce the driving frequency.
- Phase space hole and the associated BGK mode locked to the drive is formed.

Autoresonance !

The System

- Penning trapped plasma:



$\omega_d = \omega_0 - \alpha t$

- Vlasov-Poisson system:

$$\begin{cases} f_t + u f_x - \left(\varphi_x + \varphi_x^{drive}\right) f_u = 0 \\ \varphi_{xx} - \kappa^2 \varphi = \eta^2 \left(1 - \int_{-\infty}^{\infty} f\left(u,x,t\right) du\right) \end{cases}$$

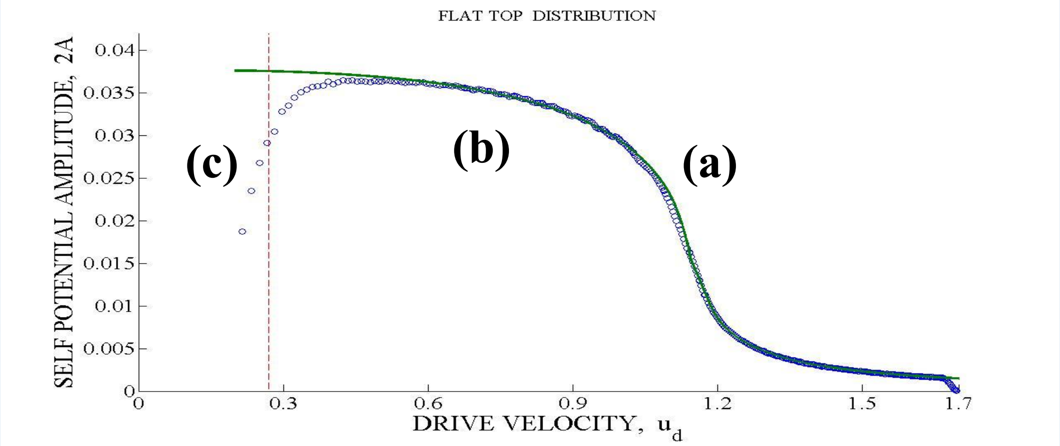
- Reflecting boundary conditions:

$$\begin{cases} f\left(u,0,t\right) = f\left(-u,0,,t\right) \\ f\left(u,1,t\right) = f\left(-u,1,t\right) \end{cases}$$

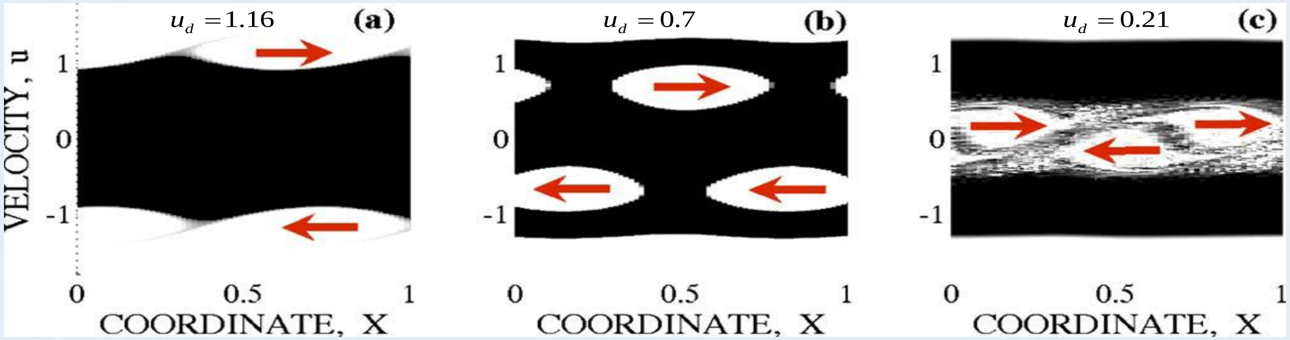
• *L. Friedland, F. Peinetti, W. Bertsche, J. Fajans, and J. Wurtele* , Phys. Plasmas 11, 4305 (2004)

Simulation Results

Electrostatic wave amplitude:



Phase space holes formation:



Holes outside

Holes inside

Collision and resonance overlap

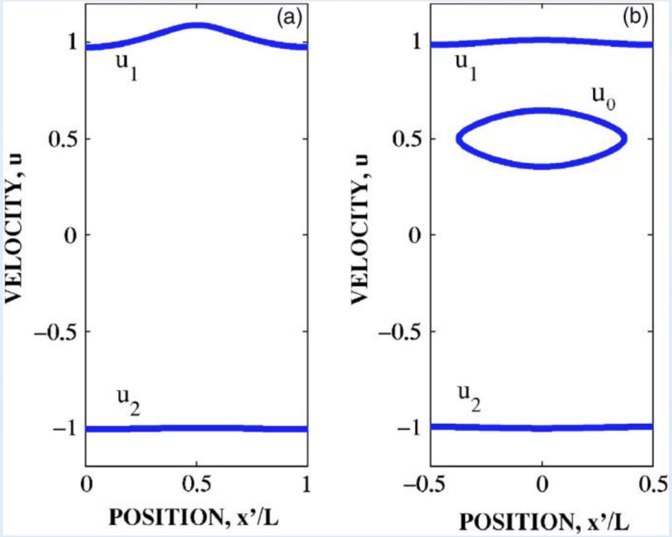
Phase space holes ⟷ BGK mode

The Water Bag model

Assumptions:

I. Plasma between limiting trajectories.
II. Perfect phase locking.

Dynamics of the limiting trajectories

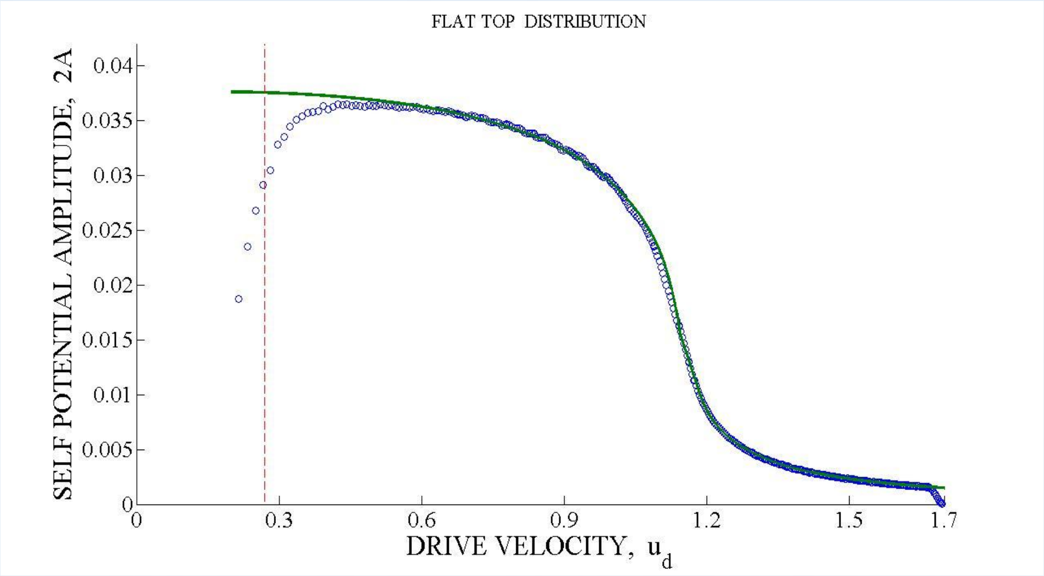


Differential equations → Algebraic equations

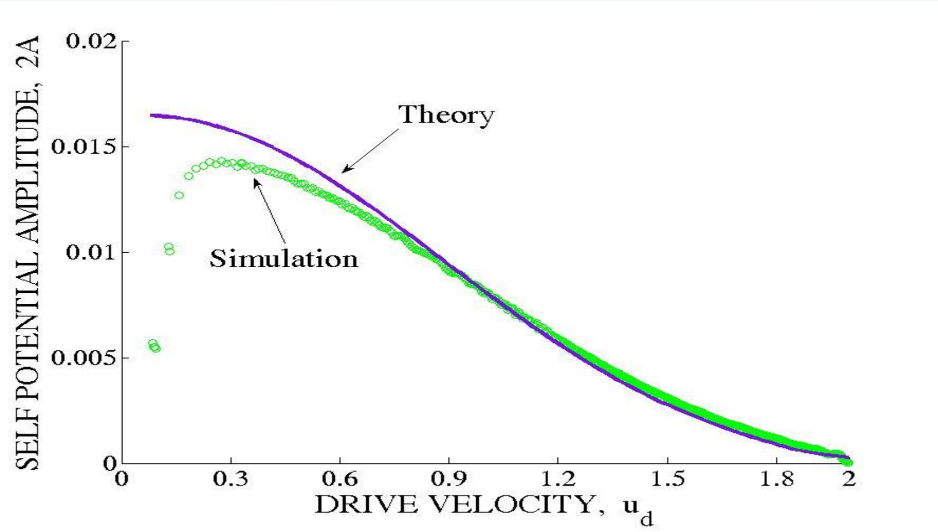
- Hamiltonian system with slow parameters.
- Each limiting trajectory has its own conserved action.
- Add Fourier transformed Poisson equation
- Sufficient number of algebraic equations to describe evolution of the limiting trajectories and the associate BGK mode.

Simulation vs. Theory

- Flat top Distribution



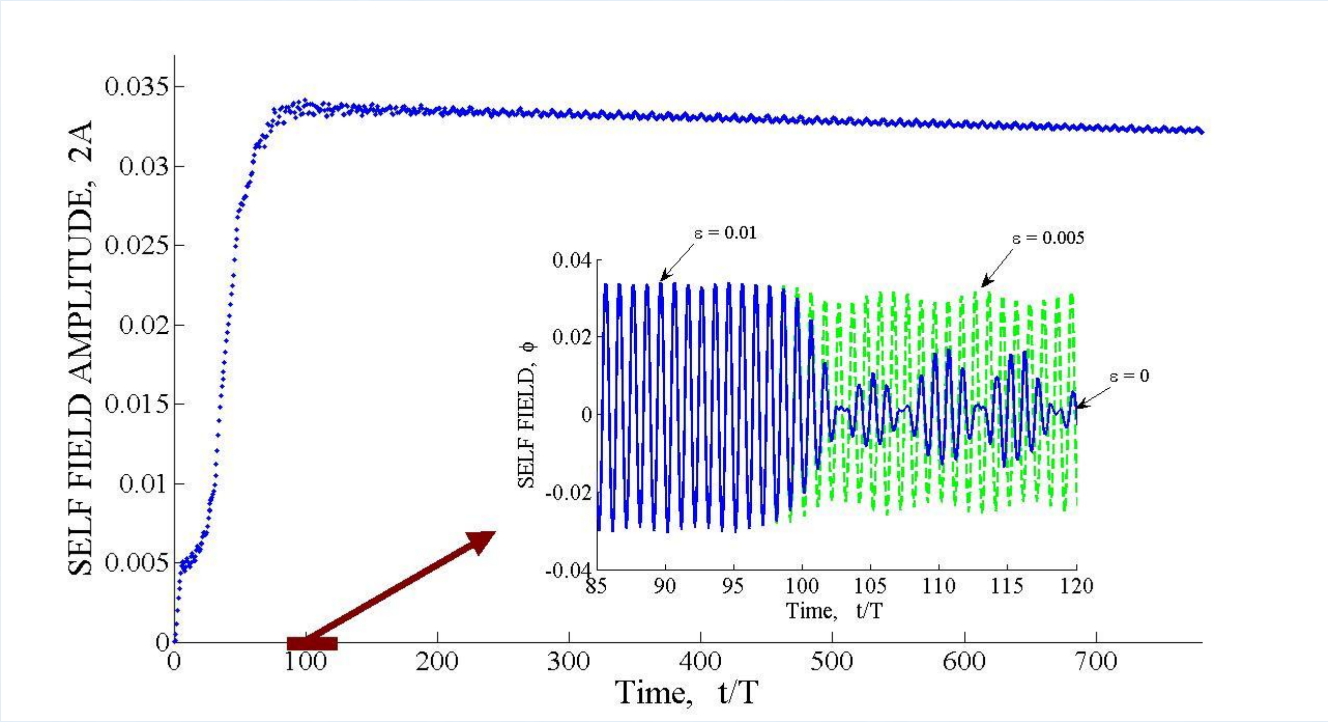
- Mexwellian Distribution



Summary

- We found a way to excite and control BGK modes in trapped single species plasmas.
- Our water bag theory fully agrees with simulations.
- Thank you !

Stability



SELF FIELD AMPLITUDE, 2A
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
100
200
300
400
500
600
700
Time, t/T
ε = 0.01
ε = 0.005
ε = 0
SELF FIELD, φ
0.04
0.02
0
-0.02
-0.04
85
90
95
100
105
110
115
120
Time, t/T

Vlasov-Poisson System

$$\begin{cases} f_t + uf_x - \left(\varphi_x + \varphi_x^{drive}\right) f_u = 0 \\ \varphi_{xx} - \kappa^2 \varphi = \eta^2 \left(1 - \int_{-\infty}^{\infty} f\left(u,x,t\right) du\right) \end{cases}$$

$\kappa =$ radial screening parameter.

$\eta =$ dencity parameter.

$$\varphi^{drive} = 2\varepsilon \cos\left(kx\right) \cos\left(\int \omega_d\left(t\right) dt\right)$$

$$\omega_d = \omega_0 - \alpha t$$

Reflecting boundary conditions: $\begin{cases} f\left(u,0,t\right) = f\left(-u,0,,t\right) \\ f\left(u,1,t\right) = f\left(-u,1,t\right) \end{cases}$

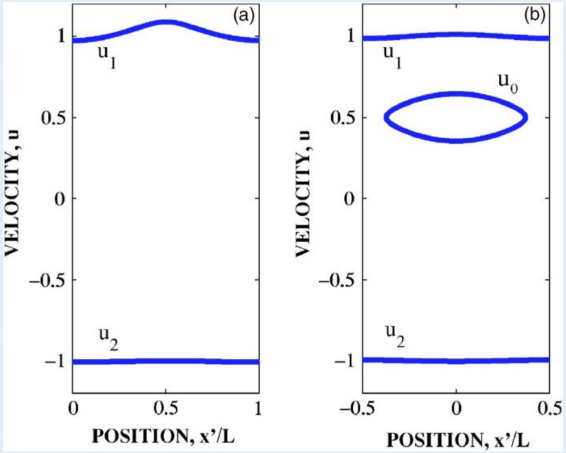
Differential equations → Algebraic equations

- Limiting trajectory Hamiltonian: $H(u,x') = \frac{1}{2}(u-u_d)^2 - A'\cos(kx')$
- Poisson equation:
  - Holes outside $A(k^2+\kappa^2) = \eta^2(F_2 - F_1)$
  - Holes inside $A(k^2+\kappa^2) = \eta^2(F_1 + F_2 + F_0)$

- Adiabatic Invariants: $J_{0,1,2} = \frac{1}{4}\int u_{0,1,2}\,dx'$

$$F_{0,1,2} = \frac{1}{2}\int u_{0,1,2}\cos(kx')\,dx'$$

$$u = u_d + \sqrt{2(H + A'\cos(kx'))}$$



- 2 free holes = 1 trapped hole

Symmetry condition:

$$f(u,x,t) = f(-u,-x,t)$$
$$\varphi(x,t) = \varphi(-x,t)$$

Adiabatic Driven Water Bag model

- Adiabatic transformation between stationary driven water bag equilibria.
- Each limiting trajectory posses an adiabatic invariant
- We get a complete set of Algebraic equations for the self field and the limiting trajectories.